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# O(6,22) BPS CONFIGURATIONS OF THE HETEROTIC STRING

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## ABSTRACT

We present a static multi-center magnetic solution of toroidally compactified heterotic string theory, which is T-duality covariant. The space-time geometry depends on the mass $M$ and on the O(6,22)-norm $N$ of the magnetic charges. For different range of parameters $(M, N)$-solution includes 1) two independent positive parameters extremal magnetic black holes with non-singular geometry in stringy frame ($a = 1$ black holes included), 2) $a = \sqrt{3}$ extremal black holes, 3) singular massive and massless magnetic white holes (repulsons). The electric multi-center solution is also given in an O(6,22)-symmetric form.

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We have found an O(6,22)-covariant (i.e. T-duality covariant) BPS multi-monopole solution, which solves the field equations of the heterotic string compactified on a 6-dimensional torus. This solution has one-half of $\mathcal{N}=4$ supersymmetry unbroken. The effective action describes the $\mathcal{N}=4$ supergravity multiplet interacting with 22 $\mathcal{N}=4$ abelian vector supermultiplets[3].

We start with the T-duality invariant bosonic action in the form of Maharana-Schwarz [1] and Sen [2],

$$\begin{aligned} S &= \frac{1}{16\pi}\int d^4x\sqrt{-\det G}\, e^{-2\phi}\Big[R_G + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{8}G^{\mu\nu}Tr(\partial_\mu\mathcal{M}L\partial_\nu\mathcal{M}L) \\ &\quad -\frac{1}{12}(H_{\mu\nu\rho})^2 - G^{\mu\mu'}G^{\nu\nu'}F^a_{\mu\nu}\,(L\mathcal{M}L)_{ab}\,F^b_{\mu'\nu'}\Big]\,. \end{aligned} \tag{1}$$

The pure magnetic solution is given by

$$\begin{aligned} &ds^2_{\rm str} = -dt^2 + e^{-4U}d\vec{x}^2\ , \qquad\qquad e^{-4U} = 2\,\chi^T L\chi = e^{4\phi}\ , \\ &\mathcal{M} = \mathbf{1}_{28} + 2e^{4U}\begin{pmatrix} A^2\,nn^T & AB\,np^T \\ AB\,pn^T & A^2\,pp^T \end{pmatrix}, \qquad\qquad \vec{H}_m = \partial_m\vec{\chi}\ . \end{aligned} \tag{2}$$

Here the magnetic potential of the theory $\vec{\chi}$ is given by the 28-dimensional harmonic $O(6,22)$-vector

$$\vec{\chi}(x) = \begin{pmatrix} \vec{\chi}_{\rm vec}(x) \\ \vec{\chi}_{\rm gr}(x) \end{pmatrix} = \begin{pmatrix} A(x)\,\vec{n} \\ B(x)\,\vec{p} \end{pmatrix}\ , \qquad \partial_i\partial_i\vec{\chi}(x) = 0\ , \tag{3}$$

$A(x)$ and $B(x)$ are harmonic functions, and $\vec{n}$ and $\vec{p}$ are arbitrary 22- and 6-dimensional unit vectors respectively. There is one vector field in each vector supermultiplet and 6 vector fields in supergravity multiplet. The vector fields of the vector multiplets are described by $\vec{\chi}_{\rm vec}$, the 22-dimensional harmonic magnetic potential. The 6-dimensional harmonic magnetic potential of vector fields from supergravity multiplet is given by $\vec{\chi}_{\rm gr}$.

The 28×28 symmetric matrix $L$ with 22 eigenvalues $-1$ and 6 eigenvalues $+1$ defines the metric in the O$(6,22)$ space. The magnetic fields $H_m^{(a)} = \frac{1}{2}\epsilon_{mij}F_{ij}^{(a)}$, $a=1,\ldots,28$, also form an O$(6,22)$ vector. The matrix $\mathcal{M}$ describes the scalar fields, and there is no axion field. The canonical metric is

$$ds^2_{\rm can} = -e^{2U}dt^2 + e^{-2U}d\vec{x}^2\ . \tag{4}$$

We can choose the harmonic functions $A$ and $B$ to get a spherically symmetric solution with asymptotically flat metric and vanishing at infinity dilaton and scalar fields $\mathcal{M}$ as

$$A(x) = \frac{P_{\rm vec}}{r}\ , \qquad B(x) = \frac{1}{\sqrt{2}} + \frac{P_{\rm gr}}{r}\ . \tag{5}$$

Here the total magnetic charge of the vector fields of the 22 vector supermultiplets is

$$P_{\rm vec} = \sqrt{(\vec{P}_{\rm vec})^2}\ , \qquad \vec{P}_{\rm vec} \equiv P_{\rm vec}\,\vec{n}\ . \tag{6}$$

[3] One could as well consider an arbitrary number of vector multiplets $n$. The symmetry of the solution would be O(6,n).

The total magnetic charge of the vector fields of the supergravity multiplet is

$$P_{\rm gr} = \sqrt{(\vec{P}_{\rm gr})^2} \ , \qquad \vec{P}_{\rm gr} \equiv P_{\rm gr}\vec{p} \ . \tag{7}$$

The supersymmetric properties of the solution are reflected in the fact that the mass is non-negative and is related to the magnetic charge of the gravitational multiplet,

$$M = |Z| = \frac{1}{\sqrt{2}} P_{\rm gr} \geq 0 \ . \tag{8}$$

Thus all our monopoles are the BPS states. The solution is defined by 28 independent parameters ($P_{\rm gr}$, $P_{\rm vec}$, $\vec{n}$, $\vec{p}$; $\vec{n}^2 = 1$, $\vec{p}^2 = 1$). The metric and the dilaton depend only on 2 independent parameters: $M$ and $P_{\rm vec}$ with $P_{\rm gr} = \sqrt{2}M$.

Since the solution is given in terms of the harmonic functions the multi-monopole solution is obtained for the case that each harmonic function has a multi-center form. In the simplest case of asymptotically flat geometry and vanishing fields we have

$$A(x) = \sum_s \frac{P^s_{\rm vec}}{|x - x_s|} \ , \qquad B(x) = \frac{1}{\sqrt{2}} + \sum_s \frac{P^s_{\rm gr}}{|x - x_s|} \ , \qquad M^s = \frac{1}{\sqrt{2}} P^s_{\rm gr} \ . \tag{9}$$

The observation crucial for the obtaining of this extremely simple multi-monopole solution is the following. The ansatz (3) allows us to solve the Bianchi identities for purely magnetic solutions keeping T-duality symmetry manifest. Many magnetic black holes which form particular cases of this solution were known before, therefore the rest is straighforward. Indeed, the metric and the dilaton are scalars with respect to O(6, 22) and therefore we could simply find them by comparison with known supersymmetric solutions. In particular, one can choose different special bases in the 6 and 22 dimensional space and check that our solution describes the S-dual of the compactified ten-dimensional supersymmetric gravitational waves, generalized fundamental strings and chiral models [3]. The compactified form of these solutions is given in [4]. This set of supersymmetric solutions has the important property that the values of the charges of the supergravity multiplet and of the vector multiplets are independent. In particular the configuration with the mass equal to the central charge of the supergravity multiplet and smaller than the charge of the vector multiplets is supersymmetric. This fact has allowed to find the massless electrically charged white holes (repulsons) [5], [6].

The $\mathcal{M}$ matrix could be fixed as well by comparison of the spherically symmetric case with the extremal limit of the solution of Sen [2]. Although he has not given an explicit expression for the magnetic fields but the asymptotic values of the fields, the matrix $\mathcal{M}$ is invariant under S-duality and can be used for comparison with our solution for $M^2 \geq \frac{1}{2}P^2_{\rm vec}$. The configurations with $M^2 < \frac{1}{2}P^2_{\rm vec}$, as already explained above, also belong to supersymmetric magnetic configurations of the heterotic string. This, however, does not follow directly from the O(7, 23) rotation of Kerr solution [2] but from the dimensionally reduced supersymmetric generalized gravitational waves [3]. The electric partners of these magnetic solutions were identified in [5], [6] with $N_L = 0$ states of the heterotic string. One can as well use the information available about the supersymmetric

spherically symmetric solutions of the action (1) presented in [7], [8] to verify the one-center case of our solutions.

Let us now discuss the spherically symmetric magnetic solution in more detail. The conformal factor of the spatial metric in the stringy frame equals

$$e^{-4U} = 2\,\chi^T L \chi = 1 + \frac{4M}{r} + \frac{2(P_{\rm gr}^2 - P_{\rm vec}^2)}{r^2}\ , \qquad \sqrt{2}M = P_{\rm gr}\ . \tag{10}$$

The causal structure of the space-time depends dramatically on the relation between the parameters of the theory, in particular on the relation between the graviphoton charge $P_{\rm gr}$ and vector multiplets charge $P_{\rm vec}$. We will use as the second parameter, besides the mass $M$, the $\mathrm{O}(6,22)$ norm of a magnetic charge

$$N \equiv \frac{1}{2} P^a L_{ab} P^b = \frac{1}{2}(P_{\rm gr}^2 - P_{\rm vec}^2)\ . \tag{11}$$

The stringy metric in this notation and the dilaton are

$$ds^2_{\rm str} = -dt^2 + \frac{4N + 4Mr + r^2}{r^2}\, d\vec{x}^2\ , \qquad e^{4\phi} = \frac{4N + 4Mr + r^2}{r^2}\ . \tag{12}$$

The scalar curvature is

$$R_{\rm str} = \frac{2(4N)^4 - 32MNr + 4(6M^2 - 4N)r^2}{(4N + 4Mr + r^2)^3}\ . \tag{13}$$

We have calculated also the square of the Ricci tensor and the square of the Riemann tensor and have found that there are no new singularities besides those in $R_{\rm str}$. It became clear recently [5], [6] that the negative norm case, specifically $N = -1$ which corresponds to $N_L = 0$ states of the heterotic string, is available as a supersymmetric configuration, solving the field equations of the effective action of the heterotic string. Only the configurations in this class admit the massless limit while the configuration remains non-trivial. To study the singularities of our solutions we have to consider separately positive, vanishing and negative norm $N$ and positive and vanishing mass $M$.

- $M > 0$, $N > 0$ *extremal supersymmetric magnetic black holes, non-singular in stringy frame.*

  The magnetic charge of the graviphoton exceeds the magnetic charge of the vector multiplets, the norm is positive. The canonical metric, when considered in a limit from non-extremal black hole, has a singular horizon. The stringy space-time, however, is completely non-singular. It is characterized by two independent positive parameters, $(M, N)$. A particular solution in this class, $N = M^2$ is the so-called $a = 1$ extremal magnetic black hole of Gibbons [9]. It is supersymmetric when embedded into pure $\mathcal{N} = 4$ supergravity without vector multiplets [10]. The absence of singularities in stringy frame for this solution was

observed previously in [11] where the configuration was referred to as a "bottomless hole". Now we have a more general solution with analogous properties which for $r \to 0$ approaches a throat with a radius proportional to the norm of the magnetic charges:

$$ds^2 \to -dt^2 + d\eta^2 + 4N\, d\Omega^2 \ , \tag{14}$$

where $d\eta^2 = 4N(d\ln r)^2$. The dilaton tells us that inside the throat the theory is in the strong coupling regime. The scalars $\mathcal{M}$ are non-singular. We have plotted the stringy scalar curvature for some values of positive $M$ and $N$, see Fig. 1.

In the electric counterpart of this solution we have a singularity at $r = 0$ in string metric but we are in the weak coupling regime. All solutions in this class are known to have the non-extremal black hole partners with the singularity covered by the horizon [2].

- $M \geq 0$, $N = 0$ *extremal supersymmetric magnetic black holes, singular in stringy frame.*

  We have plotted the values of the stringy scalar curvatures for few values of the mass, see Fig. 2. The magnetic charge of the graviphotons equals the magnetic charge of the vector multiplets. This one-parameter solution with $N = 0, M > 0$ corresponds to the $a = \sqrt{3}$ extremal limit of the magnetic black hole of Gibbons and Perry [12]. This solution is singular at $r = 0$ in the four-dimensional space in canonical as well as in the stringy frame. For the vanishing mass $M$ the solution becomes trivial.

- $M \geq 0$, $N < 0$ *supersymmetric massive and massless magnetic white holes (repulsons).*

  When the magnetic charge of the vector multiplets exceeds the graviphoton charge an additional singularity appears at $r_0 = \sqrt{2}(P_{\rm vec} - P_{\rm gr}) > 0$. We have discussed the nature of this singularity in [6] where the corresponding singularities in canonical frame were shown to have reflecting properties. For this reason such solutions cannot be called black holes. One may call them either white holes or repulsons. The curvature for the magnetic solutions in stringy frame with an additional singularity is plotted in Fig. 3. Note that the dilaton (the string coupling) becomes very small near the singularity at $r = r_0$. In the massless case the graviphoton charge vanishes, however, the charge of the vector multiplets is non-vanishing. We have a one-parameter massless monopole solution which is singular at $r_0 = \sqrt{2}P_{\rm vec} > 0$. Solutions with $N < 0$ do not correspond to the extreme limit of any known black holes.

One can easily derive the electric multi-center solutions from our multi-monopole solutions in an O$(6, 22)$-symmetric form. In the stringy frame we get

$$ds^2_{\rm str} = -e^{4U} dt^2 + d\vec{x}^2 \ , \qquad \phi = U \ , \qquad E_i^{(a)} = e^{4U} (\mathcal{M}L)_{ab}\, \partial_i \chi^b \ , \tag{15}$$

where $U$ and $\mathcal{M}$ are defined in eq. (2). The electric fields $E_i^{(a)} = F_{ti}^{(a)}$ also form an O$(6, 22)$ vector and the norm of the electric and magnetic charges are equal, $|Q| = |P|$. We will identify these electric configurations with the states in the string spectrum (for details see [2], [13]). Since our solution saturates the Bogomolny bound (8) the right-moving oscillator modes have $N_R = \frac{1}{2}$. For the left-moving part we obtain

$$N_L - 1 = \frac{1}{2}(Q^2_{\rm gr} - Q^2_{\rm vec}) = \frac{1}{2}(P^2_{\rm gr} - P^2_{\rm vec}) = N \ . \tag{16}$$

Thus, the solution (15) describes the following states:

1) $N_L = 0$, $N_R = \frac{1}{2}$ massive and massless white holes ($N = -1$); the massless configuration can be thought of as a ground state of the theory.
2) $N_L = 1$, $N_R = \frac{1}{2}$ extremal $a = \sqrt{3}$ black holes ($N = 0$).
3) $N_L \geq 2$, $N_R = \frac{1}{2}$ discrete set of extremal black holes ($N \geq 1$; for $N = M^2 = N_L - 1$ they reduce to $a = 1$ black holes).

Our magnetic configurations (2) appear now as solitons related to these elementary string states via S-duality.

It was suggested recently [14] that one can smear the singularities at $r = 0$ of the spherically symmetric backgrounds of string theory by introducing the sources at the origin. This proposal requires some further study. It may solve the problem with singularities at $r = 0$ for $M > 0, N \geq 0$ solutions in all frames, since also the dilaton would become non-singular at $r = 0$. Still the solutions with $M \geq 0, N < 0$ would have the singularity at $r = r_0 > 0$ when considered as the solutions of the four-dimensional geometry. The uplifted geometry, however, may be non-singular.

Stringy $\alpha'$-corrections are known to modify the uplifted solution, when considered as embedded into the ten-dimensional geometry. To avoid Lorentz and supersymmetry anomalies one has to supplement the configuration with some non-Abelian fields via embedding of the spin connections into the gauge group. The symmetry of the solution will become O(6,6)×G where G is some non-Abelian group, which is part of $E_8 \times E_8$ or SO(32). We hope to study these issues in future.

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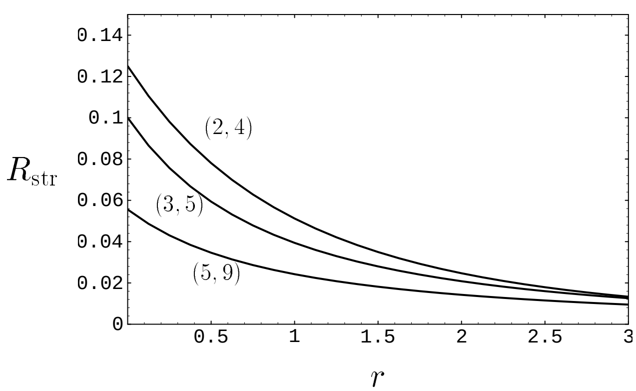


Figure 1: The stringy scalar curvature of $M > 0$, $N > 0$ monopoles as the function of $r$. Each curve is labeled with its $(M, N)$ pair. The upper curve corresponds to $a^2 = 1$ extremal magnetic black hole with $N = M^2$. All solutions have finite curvature everywhere.

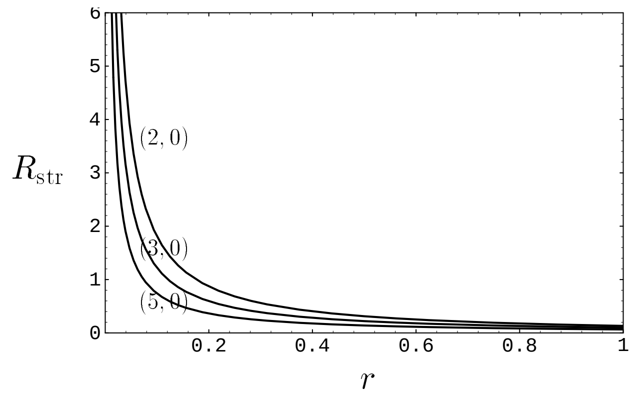


Figure 2: The stringy scalar curvature of $M > 0$, $N = 0$ monopoles ($a^2 = 3$ extremal magnetic black holes). Each curve is labeled with its $(M, 0)$ pair. The curvature is infinite at $r = 0$.

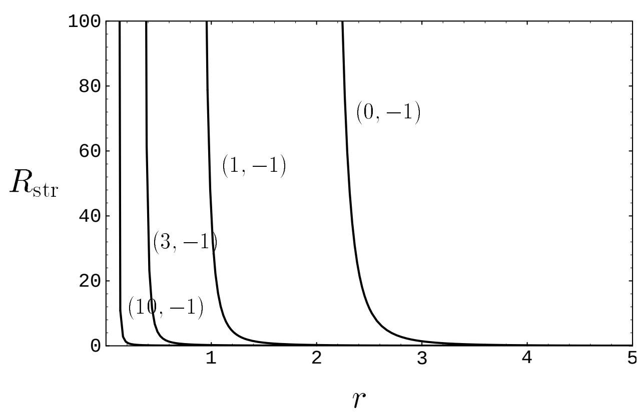


Figure 3: The stringy scalar curvature of $M \geq 0$, $N = -1$ monopoles. Each curve is labeled with its $(M, N)$ pair. The curvature is infinite at $r_0 = 2[(M^2 + 1)^{1/2} - M]$. The massless magnetic configuration (the curve (0,-1)) is singular at $r_0 = 2$.